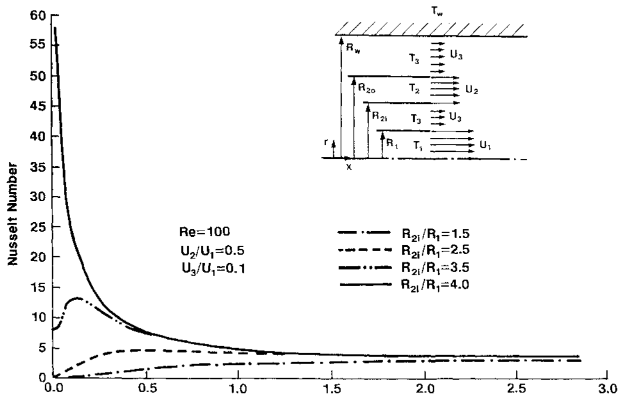

Fig. 3 Nusselt number for $Re = 100$, $U_2/U_1 = 0.5$ and $U_3/U_1 = 0.1$ and geometry.

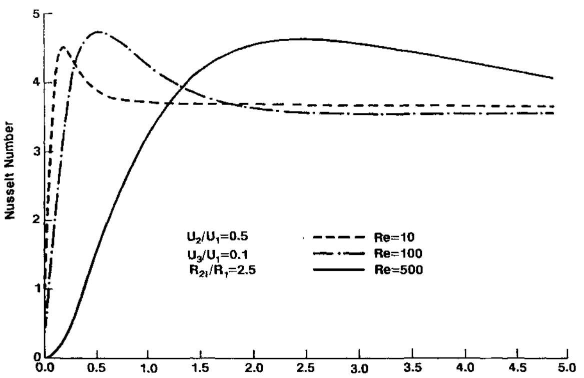

Fig. 4 Nusselt number for $U_2/U_1 = 0.5$ and $U_3/U_1 = 0.1$ and $R_{2_i}/R_1 = 2.5$.

in the Graetz problem. On one hand, an increase in the Reynolds number results in a decrease of the boundary-layer thickness along the wall, and a tendency for increased heat transfer rates. On the other hand, an increase in Reynolds number results in longer regions of very low favorable pressure gradients next to the wall. Such regions suppress the heat transfer process, therefore, resulting in lower Nusselt numbers. The combined result of these two opposing mechanisms is the behavior observed in Fig. 4.

In summary, the results demonstrate the fundamental differences between this flow and those of a simple confined jet, and of a confined jet in a single coflowing environment. Very substantial reductions in the entrance length are observed due to the addition of the third stream. Substantially higher Nusselt numbers are obtained in the three-stream case compared to the two- and single-stream cases. The distance of the intermediate stream from the pipe wall has a profound effect on the Nusselt number. The closer this stream is to the wall, the larger the Nusselt number becomes. In addition, the relative strength of the streams plays a major role in the thermal field. As the relative velocity of the middle coflowing stream is increased, the Nusselt number increases.

References

[1]Razinsky, E., and Brighton, J. A., "Confined Jet Mixing for Non-Separating Conditions," *Journal of Basic Engineering*, Vol. 93, Sept. 1971, pp. 333–349.

[2]Suzuki, K., Ida, S., and Sato, T., "Turbulence Measurements Related to Heat Transfer in an Axisymmetric Confined Jet and Laser Doppler Anemometer," *Proceedings of the 4th International Symposium on Turbulent Shear Flows*, Univ. of Karlsruhe, Karlsruhe, Germany, 1983, pp. 18.1–18.6.

[3]Khodadadi, J. M., and Vlachos, N. S., "Experimental and Numerical Study of Confined Coaxial Turbulent Jets," *AIAA Journal*, Vol. 27, No. 5, 1989, pp. 532–541.

[4]Vradis, G. C., and Otugen, M. V., "Laminar Mixing of Multiple Axisymmetric Confined Streams," 30th Aerospace Sciences Meeting, AIAA Paper 92-0369, Reno, NV, Jan. 6–9, 1992.

Analysis of Spectral Radiative Heat Transfer Using Discrete Exchange Factor Method

Yinqiu Zhang* and M. H. N. Naraghi†
Manhattan College, Riverdale, New York 10471

Introduction

In the bandwise radiative heat transfer calculations, the absorption factor of some bands is large (resembling an optically thick medium), and in others it is small (resembling an optically thin medium). Furthermore, between bands the gas is transparent, making the problem one of radiative exchange between surfaces. Those methods that are capable of solving the radiative transfer problems for a wide range of optical thicknesses, as well as nonparticipating media, has the obvious advantage for analyzing such problems. When all surfaces are at the same temperature, or the gas contains soot or other gray components, the effect of the region between bands vanishes.

In this work we present a solution technique for this multiband problem using the discrete exchange factor (DEF) method. The versatility of the DEF method in solving spectral radiative transfer problems is demonstrated by presenting three example cases with different gas and surface conditions, including surfaces at different temperatures.

Analysis

In order to use the DEF method for analysis of spectral radiative transfer in an enclosure, the nongray distribution of radiative properties is subdivided into a finite number of wavebands. It is considered that the absorption coefficient is constant within each band and the medium is transparent outside the band. The gas properties are obtained using Edwards' wide band model.[1] The direct exchange factors are evaluated at each band using the formulations given in Refs. 2 and 3. In the regions between bands, where the gas is transparent, gas-to-gas and gas-to-surface exchange factors are zero, and only surface-to-surface exchange factors must be evaluated. The direct-spectral direct-exchange factors will then be used to calculate total exchange factors between nodal points at each band. Explicit matrix formulations presented in Ref. 2 can then be used to calculate spectral total exchange factors, i.e., $(\overline{DS_iS_j})_v$, $(\overline{DS_iG_j})_v$, $(\overline{DG_iS_j})_v$, and $(\overline{DG_iG_j})_v$.

Once the spectral total exchange factors are determined, the heat fluxes and emissive powers of the surface and the gas nodes can be obtained at each band. The overall heat flux at node i is calculated by integrating the spectral heat flux distribution over all wave numbers v

$$q_i'' = \int_0^\infty q_{i,v}''\,\mathrm{d}v \tag{1}$$

This equation can be integrated numerically to yield the following discretized formulations for calculating the overall heat

Received June 25, 1992; revision received Sept. 27, 1992; accepted for publication Oct. 1, 1992.

*Graduate Student, Department of Mechanical Engineering.
†Associate Professor, Department of Mechanical Engineering.

fluxes and emissive powers of the surface and the gas nodes:

$$q''_{si} = E_{si} - \sum_{k=0}^{N_k}\sum_{j=1}^{N_s} w_{sj}\overline{DS_jS_i}_{,k}E_{sj,k} - \sum_{k=0}^{N_k}\sum_{j=1}^{N_g} w_{g,j}\overline{DG_jS_i}_{,k}E_{gj,k} \tag{2}$$

$$q''_{gi} = \sum_{k=0}^{N_k} E_{gi,k} - \sum_{k=0}^{N_k}\sum_{j=1}^{N_s} w_{sj}\overline{DS_jG_i}_{,k}E_{sj,k} - \sum_{k=0}^{N_k}\sum_{j=1}^{N_g} w_{g,j}\overline{DG_jG_i}_{,k}E_{gj,k} \tag{3}$$

where N_k is the total number of bands, and N_s and N_g denote the total numbers of surface and gas nodes, respectively. $E_{sj,k}$ and $E_{gj,k}$ are the emissive powers of surface or gas nodal point j at spectral band k and are given by

$$E_{sj,k} = \varepsilon_j\left[f\left(\frac{T_{sj}}{\nu_k}\right) - f\left(\frac{T_{sj}}{\nu_{k+1}}\right)\right] \tag{4}$$

$$E_{gj,k} = 4K_{t,k}(1 - \omega_{0,k})\left[f\left(\frac{T_{gj}}{\nu_k}\right) - f\left(\frac{T_{gj}}{\nu_{k+1}}\right)\right] \tag{5}$$

Since the surface is gray, the overall emissive power for a surface node i is $E_{si} = \varepsilon_i\sigma T_{si}^4$. The term $[f(T/\nu_k) - f(T/\nu_{k+1})]$ is the fraction of blackbody radiation between the interval of wave numbers ν_k and ν_{k+1}:

$$f\left(\frac{T}{\nu_k}\right) - f\left(\frac{T}{\nu_{k+1}}\right) = \int_{\nu_k}^{\nu_{k+1}} \frac{2\pi hc^2\nu^3}{e^{hc\nu/kT} - 1}\,d\nu \tag{6}$$

In Eq. (1), the overall radiative energy is defined as the integration of the spectral energy over wave numbers from zero to infinity. The discretized equations of the overall energy [Eqs. (2) and (3)], are expressed in terms of sum of the spectral contribution from each band. The band index k takes values from 0 to N_k. The index values from 1 to N_k denote the N_k bands in which the gas absorptivities are significant. The index $k = 0$ implies the regions between the absorption bands where the gas is transparent and its absorptivity is zero. Hence, radiation can transmit through the gas, and the spectral emissive power of surface node j at $k = 0$ is evaluated by

$$E_{sj,0} = \varepsilon_j\sigma T_{sj}^4 - \sum_{k=1}^{N_k}\varepsilon_j\left[f\left(\frac{T_{sj}}{\nu_k}\right) - f\left(\frac{T_{sj}}{\nu_{k+1}}\right)\right] \tag{7}$$

Note that there is no gas emission for $k = 0$ since gas emissivity is equal to zero and the problem reduces to radiative exchange between surfaces.

Results and Discussion

The procedure described in the previous section is used to analyze spectral radiative heat transfer for three cases. Cases one and two show comparisons with discrete-ordinates and zone method solutions, respectively, and case three presents some new results.

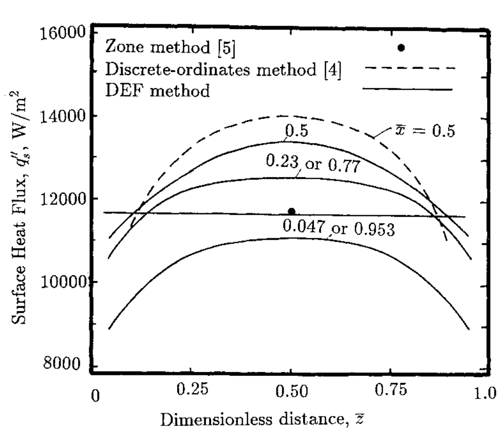

Fig. 1 Wall heat fluxes for case 1.

Case 1

To examine the accuracy of the present approach, a cubical enclosure analyzed by Fiveland[4] using the discrete-ordinates method, is used for sample calculation. The cubical enclosure with side length 0.964 m ($\sqrt{10}$ ft) consists of 10% H_2O vapor, 10% CO_2, and 80% N_2 by volume. The gas mixture is maintained at a temperature of 1367 K and a total pressure (1 atm). Surfaces of this enclosure are gray, their emissivities are 0.6, and temperatures are 1089 K. Figure 1 shows the results of the DEF method and those of discrete-ordinates method (S4)[4] and zonal method.[5] The gas properties are given

Table 1 Spectral data for case 1

Wave number, cm^{-1}		Gas	Absorption
Lower limit	Upper limit	transmissivity	coefficient, m^{-1}
410.0041	557.9734	0.2640	2.3681
557.9734	776.0361	0.0829	4.4281
776.0361	799.1050	0.2640	2.3681
944.7331	975.2293	0.9000	0.1873
1044.714	1075.385	0.9000	0.1873
1356.484	1843.318	0.5937	0.9272
2148.689	2410.219	0.1972	2.8871
3403.676	3435.246	0.7278	0.5650
3435.246	3885.004	0.5368	1.1063
3885.004	4116.921	0.7278	0.5650
5181.348	5186.722	0.9000	0.1873
5186.722	5213.765	0.8100	0.3747
5213.765	5518.764	0.9000	0.1873
7132.668	7369.197	0.9000	0.1873

Table 2 Spectral data for case 2

Wave number, cm^{-1}		Gas	Absorption
Lower limit	Upper limit	transmissivity	coefficient, m^{-1}
0	529	0.2069	1.1815
529	805	0.0477	2.2826
805	812	0.2069	1.1815
904	1004	0.9000	0.0789
1004	1016	0.8100	0.1581
1016	1116	0.8998	0.0792
1163	2037	0.5048	0.5127
2090	2410	0.1595	1.3767
3308	3347	0.5884	0.3978
3347	3729	0.2319	1.0960
3729	4212	0.5884	0.3978
5151	5197	0.8993	0.0796
5197	5249	0.6264	0.3508
5249	5503	0.6966	0.2712
7119	7381	0.7408	0.2250

Table 3 Average heat fluxes on walls for case 3

Gas total pressure, atm	Gas temperature, K	Wall heat flux, W/m^2
1	1150	2526.8
	1200	5327.4
	1250	8414.1
	1350	15482
	1550	33298
5	1150	4891.7
	1200	10327
	1250	16315
	1350	29973
	1550	64221
10	1150	5478.5
	1200	11577
	1250	18312
	1350	33828
	1550	73511

in Table 1. The grid size for the DEF method is $5 \times 5 \times 5$, which is identical to that used when applying discrete-ordinates method.[4] Since the Gaussian quadrature is used in the DEF method, the nodal points are unequally spaced. Therefore, comparison can only be made for the centerline nodes, i.e., along $\bar{x} = 0.5$, $\bar{y} = 0.5$, or $\bar{z} = 0.5$ lines. As shown in Fig. 1, the results of the DEF method are slightly different from those of discrete-ordinates method at the centerline; however, the average heat fluxes are identical and very close to those obtained using the zone method.

Case 2

Consideration is given to a black cubical enclosure with side length $L = 2$ m. One surface of the enclosure is at constant temperature of 750 K and the others are maintained at 1309.7 K. The gases mixture consists of 15% H_2O vapor, 15% CO_2, and 70% N_2. The gas temperature is 1500 K and the total pressure is 1 atm. The spectral calculation is furnished by using the DEF method with a $5 \times 5 \times 5$ grid. The mean beam length for this enclosure is taken as $2L/3$,[6] using the assumption of optically thin gases. The gas properties are determined using the wide band model[1] and are presented in Table 2. The resulting wall heat flux for the constant temperatue surface is shown in Fig. 2. The same problem was also solved by Nelson[6] using the zone method. In Nelson's solution the gas is taken as one zone. The resulting wall heat flux obtained by Nelson is 1.712×10^5 W/m^2, while the average value of wall heat flux based on the DEF method is 1.7032×10^5 W/m^2.

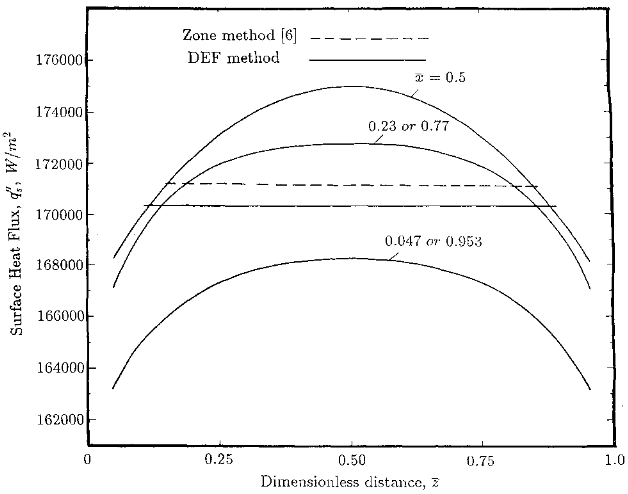

Fig. 2 Wall heat fluxes for case 2.

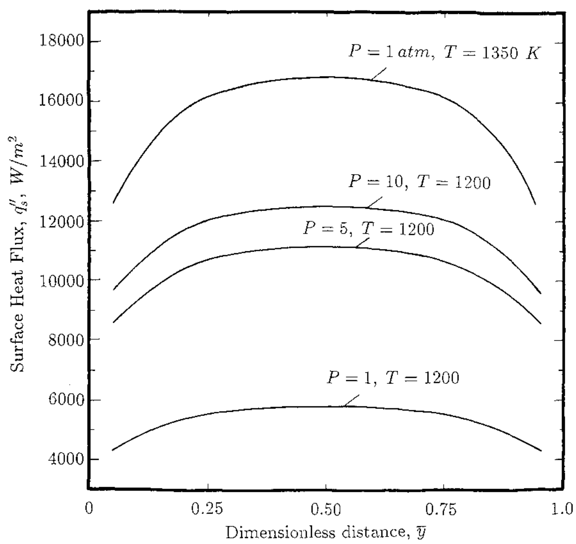

Fig. 3 Wall heat fluxes for case 3.

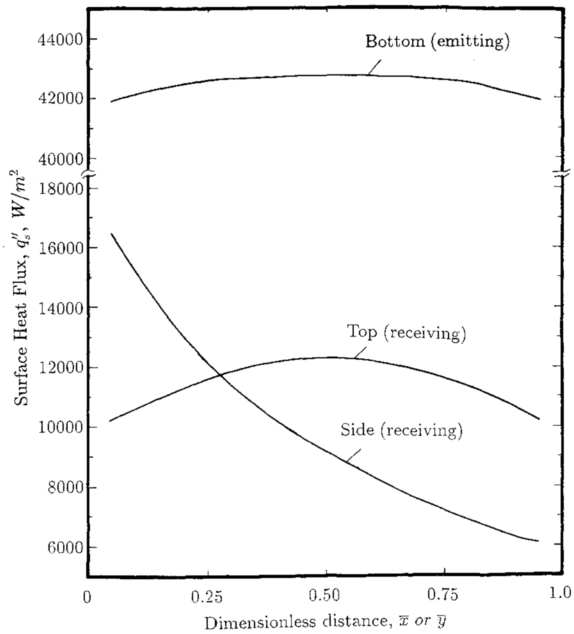

Fig. 4 Wall heat fluxes for case 3.

Case 3

In this case, a two-dimensional rectangular enclosure with a gas mixture of 15% H_2O vapor, 10% CO_2, 5% CO, and 70% N_2 is considered. Emissivities of surfaces are 0.7 and their temperatures are 1100 K. Figure 3 shows the resulting wall heat fluxes vs dimensionless distance for several different gas temperatures and total pressures. The numerical values of the average heat fluxes for different gas pressures and temperatures are listed in Table 3. Next, the same rectangular enclosure, with the exception of the gas temperature at 750 K and pressure at 1 atm, and one surface of the enclosure maintained at temperature of 1100 K and the others at 750 K is analyzed. Figure 4 shows the resulting wall heat fluxes on the bottom (1100 K), the side (750 K) and the top (750 K) walls.

Concluding Remarks

A formulation for analysis of spectral radiative heat transfer problems is presented. This formulation is based on the discrete exchange factor method and uses Edward's wide band model to obtain spectral data. The results are in excellent agreement with those of the zonal method and differ by less than 5% from those of the discrete-ordinates method. It has been demonstrated that the method is effective in analysis of enclosures with surfaces at different temperatures, where radiation exchange between surfaces in the region between band and bands with low absorption coefficient is significant.

References

[1]Edwards, D. K., "Molecular Gas Band Radiation," *Advances in Heat Transfer*, edited by T. F. Irvine and J. P. Hartnett, Vol. 12, 1976, pp. 115–193.

[2]Naraghi, M. H., Chung, B. T. F., and Litkouhi, B., "A Continuous Exchange Factor Method for Radiative Exchange in Enclosures with Participating Media," *Journal of Heat Transfer*, Vol. 110, No. 2, 1988, pp. 456–462.

[3]Naraghi, M. H. N., and Kassemi, M., "Radiative Transfer in Rectangular Enclosures: A Discretized Exchange Factor Solution," *Journal of Heat Transfer*, Vol. 111, No. 4, 1989, pp. 1117–1119.

[4]Fiveland, W. A., and Jamaluddin, A. S., "Three-Dimensional Spectral Radiative Heat Transfer Solutions by the Discrete-Ordinates Method," *Journal of Thermophysics and Heat Transfer*, Vol. 5, No. 3, 1991, pp. 335–339.

[5]Hottel, H. C., and Sarofim, A. F., *Radiative Heat Transfer*, McGraw-Hill, New York, 1967.

[6]Nelson, D. A., "Band Radiation of Isothermal Gases Within Diffuse-Walled Enclosures," *International Journal of Heat and Mass Transfer*, Vol. 27, No. 10, 1984, pp. 1759–1769.